

Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	B.Tech	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Dr. Durga Nagarajan	Slot	CD+CE+CC1
Time	3 Hours	Class No.	CH2022235002169
		Max. Marks	100

Part-1 (10 X 10 Marks)

Answer any 10 questions

- Q1. (a) Solve the following differential equation using method of variation of parameters. [10]
 $y'' - y' - 6y = 20e^{-2x}$.
- (b) Solve the nonlinear partial differential equation $q^2 - z^2p^2(1 - p^2)$. [10]
- Q2. Solve $x^2y'' - 5xy' + 8y = 8x^6$, $y(\frac{1}{2}) = 0$, $y'(\frac{1}{2}) = 0$. [10]
- Q3. Solve the linear partial differential equation $x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2 + y^2)$. [10]
- Q4. Find $L^{-1}\left(\frac{s^2 + s - 2}{s(s+3)(s-2)}\right)$. [10]
- Q5. (a) Find the Laplace transform of $f(t) = \begin{cases} t & \text{for } 0 < t < 4 \\ 5 & \text{for } t \geq 4 \end{cases}$. [10]
- (b) Solve the PDE $u_x + 2u_t = xt$, for $x > 0$, $t > 0$, given that $u(x, 0) = 0$, $u(0, t) = 0$ using Laplace transform.
- Q6. Solve $y'' + 3y' + 2y = \delta(t - 2)$, given $y(0) = 0$, $y'(0) = 0$ using Laplace transform. Here, δ corresponds to Dirac delta function. [10]
- Q7. Find the Fourier series of $f(x) = \frac{\pi - x}{2}$ in the interval $(0, 2\pi)$. Hence, deduce that [10]
 $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
- Q8. Find the half range Fourier sine series and half range Fourier cosine series for $f(x) = x$ in the interval $0 < x < 2$. [10]
- Q9. Find the Fourier transform of the function $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$. Hence, show that [10]
 $\int_0^\infty \frac{\sin s - s \cos s}{s^2} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$.
- Q10. Evaluate $\int_0^\infty \frac{1}{(x^2 + 1)(x^2 + 4)} dx$ using Fourier cosine transform. [10]
- Q11. Use Z-transform to solve $y_{n+2} + 4y_{n+1} - 5y_n = 24n - 8$ given that $y_0 = 3$, $y_1 = -5$. [10]
- Q12. (a) Evaluate $Z^{-1}\left(\frac{20z}{(z-7)(z-11)}\right)$. [10]
- (b) Find (i) $Z[nC_2]$, (ii) $Z[(n+1)(n+2)]$.



Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Sowndar Rajan P.T.	Slot	A1+TA1+IAA1
		Class Nbr	CH2022232300429
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)

Answer any 10 questions

01. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = \sin\{2\{\log(1+x)\}\}$ [10]

 02. a) Determine the charge on the capacitor at any time $t > 0$ in circuit series having an emf $E(t) = 100 \sin 60t$, a resistor of 2Ω , an inductor of $0.1H$ and capacitor of $\frac{1}{260} F$, if the initial current and charge on the capacitor are both zero. [5 marks] [10]

 b) Solve $p^2 + q^2 = x + y$ where p and q are $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively. [5 marks]

03. A heat transfer problem is described by the first-order linear partial differential equation: [10]

$$4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u.$$

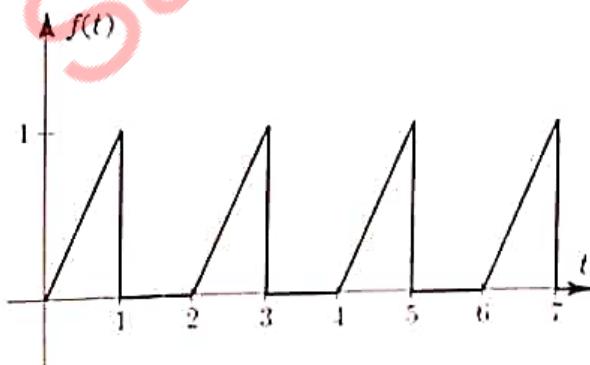
 Initially, the temperature distribution is given as $u = 3e^{-y} - e^{-5y}$ when $x = 0$. Determine the temperature distribution $u(x, y)$ for the entire domain using the method of separation of variables. [10]

04. Consider the function [10]

$$g(t) = \begin{cases} \sin t & 0 \leq t < 1 \\ t & 1 < t < 5 \\ 1 & 5 < t \end{cases}$$

 Write $g(t)$ in terms of the unit step function. Find the Laplace transform of $g(t)$.

05. a) Find the Laplace transform of the periodic function: [5 marks] [10]



Graph of periodic unit ramp function

b) Find the Fourier cosine series of the function

$$f(x) = \begin{cases} x^2 & \text{if } 0 \leq x \leq 2 \\ 4 & \text{if } 2 \leq x \leq 4. \end{cases} \quad [5 \text{ marks}]$$

06. Solve $\frac{\partial u(x, t)}{\partial t} + \frac{\partial u(x, t)}{\partial x} + 2u(x, t) = 0$ with the initial and boundary conditions [10]

$$u(0, t) = -\cos 2t \text{ and } u(x, 0) = -e^{-2x} \cos(2x) + 1.$$

07. Solve the following system of differential equations $\frac{dx}{dt} - y = e^t$ and $\frac{dy}{dt} + 4x = 2$ with the [10]

initial conditions $x = y = 0$ when $t = 0$.

08. Find the Fourier series expansion of $f(x) = x^2$ in $(-1, 1)$ and hence deduce the value of [10]

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

09. Find the Fourier sine and cosine transform of the function [10]

$$f(x) = \begin{cases} x & 0 < x < 2 \\ 3 - x & 2 < x < 3 \\ 0 & x \geq 3 \end{cases}$$

10. Find the Fourier transform of $F(e^{-x^2})$ and hence find the Fourier transform of $e^{-7(x-3)^2}$ [10]

11. Evaluate [10]

$$Z\left(\frac{1}{n(n+1)}\right) \text{ and } Z^{-1}\left(\frac{4 - 8z^{-1} + 6z^{-2}}{(1+z^{-1})(1-2z^{-1})^2}\right)$$

12. Solve the difference equation $y(n+2) - 5y(n+1) + 6y(n) = n$ using Z-transform with the [10]

initial conditions $y_0 = 1$ and $y_1 = 2$

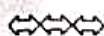
Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Abhishek Kumar Singh	Slot	A2+TA2+TAA2
		Class Nbr	CH2022232300446
Time	3 Hours	Max. Marks	100

PART-A (10 X 10 Marks)

Answer any 10 questions

- Q1. Solve the ODE $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = (\ln x) \sin(\ln x)$. [10]
- Q2. (a) Solve the ODE $\frac{d^2y}{dx^2} - y = e^x$ by using undetermined coefficient method. [5-marks] [10]
 (b) Solve $y^2p - xyq = x(z - 2y)$. [5-marks]
- Q3. (a) Solve $p^2z^2 + q^2 = p^2q$. [5-marks] [10]
 (b) Solve $p - q = \ln(x + y)$. [5-marks]
- Q4. Evaluate $L^{-1} \left[\frac{s^2 - 5s + 7}{(s + 2)^2} \right]$. [10]
- Q5. (a) Find $L[tH(t - 1) + e^{2t}\delta(t - 2)]$. [5-marks] [10]
 (Note: $H(t - a)$ is the unit step function at the point a and $\delta(t - b)$ is the impulse function at the point b .)
 (b) Find the Fourier sine series of $f(x) = \begin{cases} \frac{\pi x}{4}, & 0 \leq x < \pi/2 \\ \frac{\pi(\pi - x)}{4}, & \pi/2 \leq x < \pi \end{cases}$. [5-marks]
- Q6. Use Laplace transform to solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = x$, $u(0, t) = 0$ and $u(x, 0) = 0$ for $x > 0, t > 0$. [10]
- Q7. Use Laplace transform to solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2, \frac{dx}{dt} = -1$ at $t = 0$. [10]
- Q8. Obtain the Fourier cosine series of $x \sin(x)$ in $(0, \pi)$. Hence show that $\frac{1}{(1)(3)} - \frac{1}{(3)(5)} + \frac{1}{(5)(7)} - \dots = \frac{\pi - 2}{4}$. [10]
- Q9. Find the Fourier cosine transform of $f(x) = e^{-x}$. Hence evaluate $\int_0^\infty \frac{\cos mx}{1 + x^2} dx$ for $m > 0$. [10]
- Q10. Solve $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$, by using Fourier sine transform, where the initial and boundary conditions are $u(x, 0) = e^{-2x}, u(0, t) = 0$ for $x > 0$ and $t > 0$. [10]
- Q11. Use Z-transform to solve $u_{n+2} - 6u_{n+1} + 8u_n = 4^n$ such that $u_0 = 0$ and $u_1 = 1$. [10]
- Q12. (a) Find $Z[\sinh 3n]$. [5-marks] [10]
 (b) Find $Z^{-1} \left[\frac{z^3}{(z - 1)^2(z - 2)} \right]$. [5-marks]





KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION IS TREATED AS EXAM MAL
Answer any TEN Questions
(10 X 10 = 100 Marks)

1. ✓ Solve the differential equation $y''(x) - 2y'(x) + 2y(x) = e^x \tan x$, using the method of variation of parameters. +
2. ✓ Find the solution of $x^2 y''(x) - xy'(x) - 3y(x) = x^2 \log x$.
3. Solve $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$; $u(0, y) = 8e^{-3y}$ using method of separation of variables.
4. (i) Form the partial differential equation of all the spheres having centre lying in xy - plane and having constant radius. [5]
(ii) Solve $p^2 + q^2 = x + y$. [5]
5. ✓ Find the Laplace Transform of the periodic function $f(t) = \begin{cases} t & ; 0 < t < a \\ 2a - t & ; a < t < 2a \end{cases}$ with period $2a$. +
6. (i) Find the Inverse Laplace Transform of $\frac{1}{s^3(s^2+1)}$ by using convolution theorem. [6]
(ii) Evaluate $L\{f(t)\}$ where $f(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right) & ; t > \frac{2\pi}{3} \\ 0 & ; t < \frac{2\pi}{3} \end{cases}$ [4]
using second shifting property.
7. ✓ Solve the initial value problem $y'' + 3y' + 2y = g(t)$ with $y(0) = 0$, $y'(0) = 1$ where $g(t) = \begin{cases} 1 & ; \text{if } 0 \leq t < 1 \\ 0 & ; \text{if } t \geq 1 \end{cases}$ using Laplace Transform. 4
8. ✓ Expand $f(x)$ as Fourier series if $f(x) = \begin{cases} \pi x & ; 0 < x < 1 \\ \pi(2-x) & ; 1 < x < 2 \end{cases}$ +
Hence deduce that $\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \dots = \frac{\pi^2}{8}$.
9. ✓ Find a series of cosines of multiples of x which will represent $x \sin x$ in the interval $(0, \pi)$ and hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + \dots = \frac{\pi-2}{4}$. +

10. Find the Fourier Transform of $f(x) = \begin{cases} 1-|x| & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$ and hence find the value of

$$\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt.$$

11. Find the inverse Z-transform of $\frac{z^2}{(z-1)(z-3)}$ using convolution theorem. τ

12. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using τ
Z-transform.

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Satyaprakash Swain

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. KRITI ARYA	Slot	C1+TC1+TCC1
		Class Nbr	CH2022232300682
Time	3 Hours	Max. Marks	100

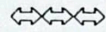
Section A (10 X 10 Marks)
Answer any 10 questions

01. Solve the ODE $2x^2y'' + 5xy' + y = \frac{x-1}{2x}$. [10]
02. (a) Find the solution of the differential equation $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0$. Hence find the Wronskian of the solutions and conclude that the solutions are linearly independent or not. [5]
 (b) Form a partial differential equation by eliminating the arbitrary function from the family of curves $z = f(x + ct) + g(x - ct)$. [5]
03. Solve the PDE: $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$. [10]
04. Find the Laplace transforms of the following functions. [10]
 (a) $f(t) = \begin{cases} \sin t, & \text{if } 0 < t < \pi \\ \sin 2t, & \text{if } \pi < t < 2\pi \\ \sin 3t, & \text{if } t > 2\pi \end{cases}$ [5]
 (b) $f(t) = e^{-5t} \int_0^t \frac{\sin t}{t} dt$. [5]
05. Solve the following partial differential equation by the method of Laplace transform [10]
 $\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial t} = -8t; x, t > 0$
 with the initial and boundary conditions $u(x, 0) = 0$ and $u(0, t) = 2t^2$.
06. Solve the following ODE using Laplace transform. [10]
 $y'' - 4y' - 5y = 30u(t - 1)$, given that $y = 0, y' = 6$, at $t = 0$ and $u(t)$ denotes the unit step function.
07. (a) Find the Laplace inverse of the function $\frac{1}{(s^2+a^2)^2}$, where a is a constant. [5]
 (b) Let $f(x) = (\pi - x)^2, x \in (0, \pi)$. Find the half range cosine series of $f(x)$ and hence find the value of $\sum_{n=1}^{\infty} \frac{1}{n^4}$. [5]
08. Let $f(x) = \begin{cases} x & 0 < x < \pi \\ 2\pi - x & \pi < x < 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$, for all real number x . Find the Fourier series of $f(x)$ and hence find the value $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$. [10]
09. Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate the Fourier sine transform of $xe^{-a^2x^2}$. [10]
10. Find the Fourier sine transform of $e^{-2x}, x > 0$ Hence evaluate $\int_0^{\infty} \frac{x^2}{(x^2+4)^2} dx$. [10]
11. (a) Find the Z-transform of the function $f(n) = ke^{-an} - 2\sin(bn) + 2^n(n^2 - n)$. [5] [10]

(b) Find the inverse Z -transform of the function $\frac{4z^2}{4z^2 - 2z\sqrt{3} + 1}$. [5]

12. Solve the following difference equation.

$$8y(n+2) + 6y(n+1) + y(n) = 5, \text{ where } y(0) = 0, y(1) = -1.$$



[10]

Satyaprakash Swain

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Berin Greeni A	Slot	C2+TC2+TCC2
		Class Nbr	CH2022232300619
Time	3 Hours	Max. Marks	100

Section A (10 X 10 Marks)

Answer any 10 questions

01. Solve by using the method of variation of parameter $4 \frac{d^2y}{dx^2} - y = \frac{1}{4}xe^{(\frac{x}{2})}$; $y(0) = 1, y'(0) = 0$. [10]

02. A) Solve the differential equation $\frac{d^2y}{dx^2} + y = 12e^{2x}$ by using the method of undetermined coefficients. [10]

B) Form a partial differential equation by eliminating f and g from the following equation $z = f(x^2 - y) + g(x^2 + y)$.

(5+5 Marks)

03. A. Solve $z^2(p^2z^2 + q^2) = 1$. Does singular solution exist?

B. Obtain the general solution of the following partial differential equation

$$\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$$

(5+5 Marks)

04. Find $L^{-1} \left[\frac{(s+1)e^{-3s}}{s^2+s+1} \right]$

a^h

$1 - \frac{\pi}{2n}$ 30

2^h

(π, π)

$2 \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$\frac{1}{s^3}$

[10]

[10]

05. A. Find the Laplace transform of $f(t) = \begin{cases} t & \text{if } t < 6 \\ -8 + (t-6)^2 & \text{if } t \geq 6 \end{cases}$ [10]

$\frac{\sqrt{3}}{2} \frac{2}{\sqrt{3}} \frac{1}{2}$

B. Using the Fourier series of $f(x) = x$ in the interval $(0, 2\pi)$, show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

(5+5 Marks)

06. Solve the following differential equation using Laplace transform $y'' + 3y' + 2y = \delta(t-1)$ [10] with the initial condition $y(0) = y'(0) = 0$, where $\delta(t-1)$ is the unit impulse at time $t = 1$.

07. Find the bounded solution $y(x, t), x, t > 0$ of following partial differential equation by the method of Laplace transform $\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-t}; x, t > 0$ with $y(x, 0) = 0$. [10]

08. Find the half range sine series for $f(x) = \begin{cases} x & 0 \leq x < \pi/2 \\ \pi - x & \pi/2 \leq x \leq \pi \end{cases}$ [10]

$4n^2 t$

Deduce (i) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

(ii) $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots = \frac{\pi^4}{96}$.

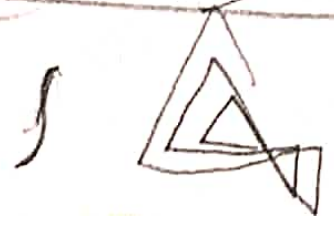
09. Find $f(x)$, if its Fourier sine transform is $\frac{e^{-ax}}{\omega}$. Hence, deduce $F_s^{-1}(\frac{1}{\omega})$. [10]

10. Find the Fourier transform of $f(x) = \begin{cases} 4 - |x| & \text{if } |x| < 4 \\ 0 & \text{if } |x| > 4 \end{cases}$. Hence show that $\int_0^\infty (\frac{\sin t}{t})^4 dt = \frac{\pi}{3}$. [10]

11. A. Find the Z-transform and the radius of convergence of $f(n) = 2^n, n < 0$ [10]

B. If $U(z) = \frac{2z^2 - 5z + 14}{(z-1)^4}$, then evaluate u_2 and u_3 .

(5+5 Marks)



12. Solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$ using Z -transform.



Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. PROSENJIT	Slot	D1+TD1+TDD1
		Class Nbr	CH2022232300628
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

01. a) Solve the differential equation $\frac{d^2y}{dx^2} + 16y = 8e^x$ by method of undetermined coefficients. [5 marks] [10]
- b) Find the singular integral of $z = px + qy - 2\sqrt{pq}$. [5 marks]
02. An LCR circuit connected in series has $R = 10$ ohms, $C = 10^{-2}$ farad, $L = \frac{1}{2}$ henry and an applied voltage $E = 12$ volts. Assuming no initial current and no initial charge at $t = 0$ when the voltage is first applied. Find the subsequent current in the system at any time t without finding the charge. [10]
03. a) Solve by using Lagrange multiplier method $y^2p + x^2q = x^2y^2z^2$. [7 marks] [10]
- b) Form a partial differential equation by eliminating the arbitrary constants a and b from $z = a(x + y) + b$. [3 marks]
04. Find the Laplace transform of $f(t) = tu(t-2) + \delta(t - \frac{\pi}{2})\cos t + \int_0^t \frac{\sin t}{t} dt$, where $u(t-2)$ and $\delta(t - \frac{\pi}{2})$ represent Unit step function and Impulse function respectively. [10]
05. Solve the given partial differential equation by using Laplace transform $\frac{\partial y}{\partial x} + x \frac{\partial y}{\partial t} = 0$ with boundary conditions $u(0, t) = t$ and $u(x, 0) = 0$. [10]
06. Solve the given differential equation using Laplace transform $\frac{d^2y}{dt^2} - \frac{dy}{dt} = \cos(2t) + \cos(2t - 12)u(t - 6)$ with the boundary conditions $y(0) = 0$ and $y'(0) = 0$. [10]
07. Find the half range Fourier cosine series of $f(x) = (\pi - x^2)$ in the interval $(0, \pi)$. Hence find the sum of the infinite series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$ by using Parseval's identity. [10]
08. a) Determine the Fourier series of the periodic function $f(x) = x - 1$ defined on $(-\pi, \pi)$. [5 marks] [10]
- b) Using convolution theorem find $f(t)$ if $L(f(t)) = \frac{1}{s(s^2+1)}$. [5 marks]
09. Evaluate $\int_0^\infty \frac{dx}{(x^2+4)(x^2+1)}$ using Fourier transform. [10]
10. Find the Fourier transform of $e^{-2|x|}$ and hence evaluate [10]

$$\int_0^\infty \frac{dx}{(x^2+4)^2}$$

11. a) Find $Z[(n+1)^2 e^{-5n+7}]$. [5 marks] [10]
- b) Find $Z^{-1}\left[\frac{s^2}{(s-0.5)(s+3)}\right]$ using convolution theorem. [5 marks]
12. Solve $y_{n+2} - 3y_{n+1} + 2y_n = 3^n + 3$ with $y_0 = 1$ and $y_1 = 1$. [10]



VIT
Vellore Institute of Technology

Reg. No. :

22BME1062

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Nathiya N	Slot	D1+TD1+TDD1
		Class Nbr	CH2022232300594
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)

Answer any 10 questions

01. a) Solve the differential equation $\frac{d^2y}{dx^2} + 16y = 8e^x$ by method of undetermined coefficients. [5 marks] [10]
b) Find the singular integral of $z = px + qy - 2\sqrt{(pq)}$. [5 marks]
02. An LCR circuit connected in series has $R = 10$ ohms, $C = 10^{-2}$ farad, $L = \frac{1}{2}$ henry and an applied voltage $E = 12$ volts. Assuming no initial current and no initial charge at $t = 0$ when the voltage is first applied. Find the subsequent current in the system at any time t without finding the charge. [10]
03. a) Solve by using Lagrange multiplier method $y^2p + x^2q = x^2y^2z^2$. [7 marks] [10]
b) Form a partial differential equation by eliminating the arbitrary constants a and b from $z = a(x + y) + b$. [3 marks]

04. Find the Laplace transform of $f(t) = tu(t-2) + \delta(t - \frac{\pi}{2})\cos t + \int_0^t \frac{\sin t}{t} dt$, where $u(t-2)$ and $\delta(t - \frac{\pi}{2})$ represent Unit step function and Impulse function respectively. [10]
05. Solve the given partial differential equation by using Laplace transform $\frac{\partial u}{\partial x} + x \frac{\partial u}{\partial t} = 0$ with boundary conditions $u(0, t) = t$ and $u(x, 0) = 0$. [10]
06. Solve the given differential equation using Laplace transform $\frac{d^2 y}{dt^2} - \frac{dy}{dt} = \cos(2t) + \cos(2t - 12)u(t - 6)$ with the boundary conditions $y(0) = 0$ and $y'(0) = 0$. [10]
07. Find the half range Fourier cosine series of $f(x) = (\pi - x^2)$ in the interval $(0, \pi)$. Hence find the sum of the infinite series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$ by using Parseval's identity. [10]
08. a) Determine the Fourier series of the periodic function $f(x) = x - 1$ defined on $(-\pi, \pi)$. [5 marks] [10]
 b) Using convolution theorem find $f(t)$ if $L(f(t)) = \frac{1}{s(s^2+1)}$. [5 marks]
09. Evaluate $\int_0^\infty \frac{dx}{(x^2+4)(x^2+1)}$ using Fourier transform. [10]
10. Find the Fourier transform of $e^{-2|x|}$ and hence evaluate [10]

$$\int_0^\infty \frac{dx}{(x^2 + 4)^2}$$

11. a) Find $Z[(n+1)^2 e^{-5n+7}]$. [5 marks]

b) Find $Z^{-1}\left[\frac{z^2}{(z-0.5)(z+3)}\right]$ using convolution theorem. [5 marks]

12. Solve $y_{n-2} - 3y_{n+1} + 2y_n = 3^n + 3$ with $y_0 = 1$ and $y_1 = 1$.

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Manimaran J	Slot	D2+TD2+IDD2
		Class Nbr	CH2022232300626
Time	3 Hours	Max. Marks	100

Section-A (10 X 10 Marks)

 Answer any 10 questions

01. Consider an LRC circuit where $L = 1$, $R = 2$ and $C = 1$. The current $I(t)$ is driven by an electromagnetic force $2 \sin 2t$. The circuit equation for the voltage $V(t)$ across the capacitor is $\frac{d^2V}{dt^2} + 2 \frac{dV}{dt} + V = 2 \sin 2t$ with $V(0) = 0$ and $V'(0) = 0$. Describe the subsequent voltage oscillations by using the method of undetermined coefficients. [10]
02. (a) Consider the initial value problem $\frac{d^2y}{dx^2} - 6y = 0$ with $y(1) = \alpha$ and $y'(1) = 6$. If $y(x) \rightarrow 0$ as $x \rightarrow 0$, then find the value of α . [10]
 (b) Solve the PDE: $py + qx = xyz^2(x^2 - y^2)$.
03. Find the complete and singular integrals of the PDE: $z = px + qy + \sqrt{1 + p^2 + q^2}$. [10]
04. Find the Laplace transform of $f(t) = u(t - 2\pi) \sin t + t^4 \delta(t - 3) + \frac{\sin t}{t} + \sqrt{t}$. [10]
05. (a) Using convolution theorem find $f(t)$ if $\mathcal{L}\{f(t)\} = \frac{9}{s(s+3)^2} e^{os}$. [10]
 (b) Evaluate the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ from the Fourier series expansion of $f(x) = x$ in the interval $(-\pi, \pi)$.
06. Solve the simultaneous equations $\frac{dx}{dt} - y = e^t$ and $\frac{dy}{dt} + x = \sin t$ with $x(0) = 1$ and $y(0) = 0$. [10]
07. Using the Laplace transform method solve the following PDE: $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ with $u(x, 0) = 6e^{-3x}$ and solution being bounded for $x > 0$, $t > 0$. [10]
08. Find the Fourier series expansion of the following periodic function in the interval $(-\pi, \pi)$ with period 2π :

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$
 Hence, obtain the value of $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. [10]
09. Find the Fourier transform of the function $f(x) = e^{-|x|}$, $-\infty < x < \infty$. Hence, (a) derive that $\int_0^{\infty} \frac{\cos \omega x}{1-\omega^2} d\omega = \frac{\pi e^{-|x|}}{2}$; and (b) obtain the Fourier transform of $x e^{-|x|}$. [10]
10. Using Fourier transform, solve the PDE $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$, subject to the conditions: (i) $u(0, t) = u_0$, $t > 0$; (ii) $u(x, 0) = 0$, $x > 0$; (iii) u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \rightarrow \infty$. [10]
11. (a) Find the Z-transform of $2^n \cos(n+3)$. [10]
 (b) Find the inverse Z-transform of $\frac{2z^2-3z}{z^2-6z+8}$.
12. Solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$ and $u_1 = 1$ by using Z-transform. [10]

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