

Q.No.	Sub. Sec.	Question Description	Marks
1.		Expand $f(x, y) = e^x \sin y$ in Taylor Series about $\left(1, \frac{\pi}{2}\right)$ up to third degree terms and hence find the approximate value of $e^{0.5} \sin\left(\frac{\pi+2}{2}\right)$.	[10]
2.		Find the absolute extrema of $z = 2x^2 + y - 3xy$ in the plane region D bounded by the lines $y = 1 - x$, $y = 1 + x$, $y = -1 - x$ and $y = -1 + x$.	[10]
3.		Sketch the region of integration and evaluate $\int_0^3 \int_{4y/3}^{\sqrt{25-y^2}} x \, dx \, dy$ after changing the order of integration.	[10]
4.	[a]	Using spherical coordinates, evaluate $\iiint_E (x^2 + y^2) \, dv$ where E lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.	[7]
	[b]	Evaluate $\iiint_R (x - y - z) \, dx \, dy \, dz$, where $R : 1 \leq x \leq 2; 2 \leq y \leq 3; 1 \leq z \leq 3$.	[3]
5.	[a]	If n is positive integer and $m > -1$, then prove that $\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$	[5]
	[b]	Using Beta and Gamma function, evaluate $\int_0^a y^4 \sqrt{a^2 - y^2} \, dy$.	[5]

Answer all the Questions

1.	<p>(i) Find all critical points of the function $f(x, y) = x^4 + y^4 - 2x^2 - 2y^2 + 4xy$ and check whether the function attains maximum or minimum at each of these points.</p> <p>(ii) Show that point $(0, 0)$ is neither a point of local minimum nor a point of local maximum for the function given by $f(x, y) = 3x^4 - 4x^2y + y^2$ for $(x, y) \in \mathbb{R}^2$.</p>	10
2.	<p>(i) If x, y and z are positive real numbers, then find the minimum value of function $x^2 + 8y^2 + 27z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.</p> <p>(ii) Find the Taylor series expansion of $f(x, y) = \sin xy + x^2y + e^x$ in the power of $(x - 1)$ and $(y - \pi)$ up to second degree terms.</p>	10

3.	<p>(i) Find the value of integral by using the polar coordinates.</p> $I = \iint_D \sqrt{x^2 + y^2} \, dydx \quad \text{where } D = \{(x, y) \in \mathbb{R}^2 : x \leq x^2 + y^2 \leq 2x\}$ <p>(ii) Find the value of integral by changing the order of integration</p> $I = \int_0^4 \int_{(4-x)^{\frac{1}{2}}}^2 e^{y^3} \, dydx$	10
4.	Using multiple integrals, find the volume of the solid region bounded above by hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$.	10
5.	<p>Solve the following integrals by using Beta and Gamma Function:</p> <p>(i) $I = \int_0^{\infty} \frac{e^{-\frac{k}{x^2}}}{x^6} dx \quad \text{where } k \neq 0$</p> <p>(ii) $I = \int_0^1 x^4 \sqrt{1-x^2} dx$</p>	10

**VIT**Vellore Institute of Technology
(Approved for the University Grants Commission, UGC, 1986)**Continuous Assessment Test (CAT)- II- December 2022**

Programme	: B.Tech.	Semester	: Fall Semester Year I 2022-2023
Course Title	: Calculus	Code	: BMAT101L
		Slot	: E2+TE2
Faculty	: Dr. Berin Greeni A, Dr. Prosenjit Paul, Dr. Srutha Keerthi B, Dr. Dhivya P, Dr. Saurabh Chandra Maury, Dr. Karan Kumar Pradhan	Class No.	: CH2022231700201, 197, 199, 202, 198, 200
Duration	: 1 ½ Hours	Max. Marks	: 50

Answer all the Questions (50 marks)

Q. No.	Question Description	Marks
1.	(a) Find the Taylor series expansion of x^y about (1, 1) up to second degree terms. (b) Obtain the critical points of $(x^2 + 3y^2)e^{-(x^2+y^2)}$.	[5+5]
2.	A wire of length L is cut into two parts (not necessarily equal) which are bent to in the form of a square and a circle respectively. Find the least value of the sum of areas so found.	[10]
3.	If $f(x, y) = (x^2 + y^2)$ represents the population density of a planar region on the Earth, where x and y are measured in miles, find the number of people in the region shown below.	[10]
		[10]
4.	Evaluate $\iiint_R (x^2 + y^2 + z^2) dV$, where R is the region above the xy-plane bounded by the cone $z^2 = 3(x^2 + y^2)$ and by the sphere $x^2 + y^2 + z^2 = 1$.	[10]
5.	Using special functions compute the integral $\iint_R x^2 y^2 dx dy$, where R is the region bounded by the curve $x^{2/3} + y^{2/3} - 4 = 0$.	[10]



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(Chartered to be a University under section 3 of UGC Act, 1956)

Continuous Assessment Test (CAT)- II- December, 2022

Programme	: B.Tech.	Semester	: Fall Semester I 2022-2023
Course Title	: Calculus	Code	: BMAT101L
		Slot	: E1+TE1
Faculty	: Dr. Saroj Kumar Dash, Dr. Manivannan A, Dr. C. Rajivganthi, Dr. Harshavarthini, Dr. P. S. S. S. S. Dr. Ashis Bera, Dr. Ankit Kumar, Dr. Sandip Saha, Dr. Kriti Arya	Class Nbr	: CH202223170 189, 191, 192, 194, 257, 323, 8
Duration	: 1 ½ Hours	Max. Marks	: 50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at the point (x, y, z) on the probe's surface is $T = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's.	[10]
2.	Find the absolute maximum and minimum of $f(x, y) = x + y - xy$ on the triangle ABC with vertices A (0,50), B(50,0) and C(-50,-50).	[10]
3.	a) Change the order of integration, evaluate $\int_0^2 \int_0^{9-x^2} x \, dy \, dx$.	[7]
	b) Find the area of $r = \sin \theta$ in polar coordinates.	[3]
4.	A spherical tank of radius 3 meters is filled with water to a height of 2 meters. Find the volume of the water using the cylindrical coordinates.	[10]
5.	a) Evaluate $\int_0^\infty \sqrt{x} e^{-x^5} \, dx$.	[5]
	b) Evaluate $\int_0^1 x^7 (1-x^2)^6 \, dx$.	[5]



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Programme	: B.Tech.	Semester	: Fall 2022-2023
Course Title	: Calculus	Code	: BMAT101L
		Slot	: A2+TA2
Faculty	: Dr. Balamurugan, Dr. Saroj Kumar Dash, Dr. Mini Ghosh, Dr. Manimaran, Dr. Sowndarrajan, Dr. Prabhakar, Dr. Rajesh Kumar, Dr. Soumendu Roy	Class ID	: CH2022231700410, 416,429,440,443,57 3,604,610
		Duration	: 1 ½ Hours
		Max. Marks	: 50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	(a). Find the absolute maximum and minimum values of $f(x, y)$ on the region R where $f(x, y) = x^2 + xy + y^2 - 3x + 3y$ and R is the triangular region cut from the first quadrant by the line $x + y = 4$.	[5]
	(b). For what values of the constant k does the second derivative test guarantee that $f(x, y) = x^2 + kxy + y^2$ will have a saddle point at $(0, 0)$? A local minimum at $(0, 0)$? For what values of k is the second derivative test inconclusive? Give reasons of your results.	[5]
2.	(a). Find three positive numbers whose sum is 50 and whose product is maximum.	[5]
	(b). A flat circular plate has the shape of the region $x^2 + y^2 \leq 4$. The plate, including the boundary where $x^2 + y^2 = 4$, is heated so that the temperature at the point (x, y) is $T(x, y) = x^2 + 2y^2 - x$. Find the temperatures at the hottest and coldest points on the plate.	[5]
3.	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(1+e^y)\sqrt{1-x^2-y^2}} dy dx$.	[10]
4.	Find the volume of the solid bounded by the xy -plane, the paraboloid $2z = x^2 + y^2$ and cylinder $x^2 + y^2 = 4$.	[10]
5.	(a). Find the value of $\int_0^\pi \sin^2 x (1 + \cos x)^4 dx$ in terms of gamma function.	[5]
	(b). Evaluate the integral $\int_0^\infty x^4 e^{-x^b} dx$.	[5]

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Continuous Assessment Test II – December 2022

Programme	: B.Tech	Semester	: FALLSEM 2022-23
Course	: Calculus	Code	: BMAT1011
Faculty	: Dr. S. Radha Dr. N. Nathiya Dr. Sowndarrajan P T Dr. Manoj Kumar Singh Dr. Harshavarthini Shanmugam Dr. Manimaran J	Slot	: AI+TA1
		Class Number	: CH2022231700297 CH2022231700423 CH2022231700424 CH2022231700298 CH2022231700617 CH2022231700608
Time	: 1½ hours	Max. Marks	: 50

Answer ALL the Questions (5 x 10 = 50 marks)

Q.No.	Sec	Question Description	Marks
1.	a.	Find the critical points of the function $f(x, y) = x^3 + y^3 - 12x - 6y + 40$. Test each of these for maximum and minimum.	5
	b.	Use Taylor's formula to find a quadratic approximation of $f(x, y) = xe^y + 1$ at (1, 4).	5
2.		The temperature T at any point (x, y, z) in space is $T = 625xzy^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.	10
3.		Find the volume of the region using double integration which lies under the paraboloid $z = 4 - x^2 - y^2$ and above the disk $(x - 1)^2 + y^2 = 1$ on the xy -plane.	10
4.		Evaluate $\iiint e^{-x^2-z^2} dV$ where R is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 9$ with $1 \leq y \leq 5$ and $z \leq 0$.	10
5.	a.	Evaluate $\int_0^1 x^{\frac{7}{2}} (1 - x^{\frac{3}{2}})^{11} dx$.	5
	b.	Evaluate $I = \int_0^{\infty} x^4 e^{-x^4} dx$.	5