VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test I – September 2022

Programme	: B.Tech.	Semester	:	Fall 2022 – 23
Course Title	: Linear Algebra and Transform Techniques	Code	:	MAT3008
		Slot	:	E1+TE1
Faculty (s)	: Dr. Poulomi De, Dr. Sushmitha P	Max. Marks	:	50
Class Nbr.	: CH2022231001856, CH2022231001859	Time	:	90 minutes

Answer all the Questions (5X10=50)

Q.No.	Sub. Sec.	Question Description	Marks
1.		Solve the system of linear equations $x+y+z=1; 4x+3y-z=6; 3x+5y+3z=4$ by using LU factorization.	10
2.		Find inverse of $A=\begin{bmatrix}1 & 3 & -4\\ 1 & 5 & 1\\ 3 & 13 & -6\end{bmatrix}$ using Gauss Jordan Method.	10
3.	a	Express the vector (1,7,-4) as linear combination of u=(1,-3,2) and v=(2,-1,1) in the vector space V_3 of R.	5
	b	Determine k so that the vectors (1,2,1), (k,1,1) and (1,1,2) are linearly independent.	5
4		Find the basis of R^4 containing the vectors (1,2,-1,1) and (0,1,2,-1). Hence find dimension.	10
5.		Find the basis of row space of $A=\begin{bmatrix}1 & 3 & 1 & -2 & -3\\ 1 & 4 & 3 & -1 & -4\\ 2 & 3 & -4 & -7 & -3\\ 3 & 8 & 1 & -7 & -8\end{bmatrix}$ Hence verify Rank Nullity Theorem.	10

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Reg. No.:	
Name :	

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Continuous Assessment Test II – October 2022

Programme	: B.Tech.			
Course Title	: Linear Algebra and Transform Techniques	Semester	:	Fall 2022 – 23
		Code	:	MAT3005
Faculty (s)	: Dr. Poulomi De, Dr. Sushmitha P	Slot	:	E1+TE1
Class Nbr.	: CH2022231001856, CH2022231001859	Max. Marks	:	50
		Time	:	90 minutes

Answer all the Questions (5X10=50)

Q.No.	Sub. Sec.	Question Description	Marks
1.	a.	If $T: V_3 \to V_2$ and $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$, then show that T is a linear transformation.	7
	b.	Consider the mapping $F: R^3 \to R^2$ defined by $F(x, y, z) = (xz, y^2)$. Find F (5,-2,3) and $F^{-1}(0,0)$.	3
2.		Let T be the linear operator on $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (-2x_1 + x_2, -x_1 + 2x_2 + 4x_3, 3x_1 + x_3)$. Find the matrix of T in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where $\alpha_1 = (-1,2,1)$, $\alpha_2 = (2,1,1)$ and $\alpha_3 = (1,0,1)$.	10
3.		Apply the Gram-Schmidt process to the vectors (1,0,1),(1,0,-1),(0,3,4) to find an orthonormal basis for $V_3(R)$ with the standard product.	10
4.		Consider the following polynomials in P(t) with the inner product $< f,g >= \int_0^1 f(t)g(t)dt$ and $< f,h >= \int_0^1 f(t)h(t)dt$ where $f(t) = t + 2$, $g(t) = 3t - 2$ and $h(t) = t^3 - 2t - 3$ a) Find $< f,g >$ and $< f,h >$ b) Find $\|f\|$ and $\|g\|$ c) Normalize f and g.	10
5.		Let $F: R^4 \to R^3$ be the linear mapping defined by $F(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$. Find basis and dimension of (a) Image of F and (b) Kernel of F.	5+5

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VIT

Reg. No.: 21MIA1103

Final Assessment Test (FAT) – November/December 2022

Programme	**M.Tech. (Integrated)**	Semester	**Fall Semester 2022-23**
Course Title	**LINEAR ALGEBRA AND TRANSFORM TECHNIQUES**	Course Code	**MAT3008**
Faculty Name	**Prof. Poulomi De**	Slot	**E1+TE1**
		Class Nbr	**CH2022231001856**
Time	**3 Hours**	Max. Marks	**100**

Part A (10 X 10 Marks)
Answer <u>any 10</u> questions

1. Find the QR-decomposition of the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, where Q is an orthogonal matrix and R is an upper triangular matrix. Using this decomposition, solve $Ax = b$ for $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. [10]

2. Find the inverse of the matrix $A = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$ using the Gauss-Jordan method. What is the determinant of A^{-1}? [10]

3. Verify whether the following sets are subspaces or not. Justify your answers. [10]
   1. $W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_2 a_3 = 0\}$.
   2. $W_2 = \{(b_1, b_2, b_3, b_4) \in \mathbb{R}^4 : b_1 = b_3 = b_2 + b_4\}$.

4. Let $W = span(S)$, where S is as given below. Determine whether the given sets S form a basis for W or not. [10]
   1. $S = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \right\}$.
   2. $S = \{(1,0,0,1,-1), (0,1,2,0,1), (-1,-1,0,1,1), (4,3,4,3,-2)\}$.

5. Find a basis for the row space and column space of $A = \begin{pmatrix} 3 & 0 & 3 & 3 & 3 \\ 3 & -1 & 2 & 4 & 1 \\ 5 & 4 & 9 & 1 & 13 \\ 7 & 6 & 13 & 1 & 19 \end{pmatrix}$. Hence find the dimension of the null space of A. [10]

6. Let $T : \mathcal{M}_{2\times2}(\mathbb{R}) \to \mathcal{M}_{2\times2}(\mathbb{R})$ be defined by $T(A) = PA$, where $P = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$. Find the matrix of the linear transformation T with respect to the ordered basis $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \right\}$ for both the domain and co-domain. Note that $\mathcal{M}_{2\times2}(\mathbb{R})$ denotes the set of all 2×2 matrices with real entries. [10]

7. Is the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ defined by $T(x, y) = (x - y, -x + y, 0, 2x - 2y)$ invertible? Find a basis for the range and kernel of T. [10]

8. With proper justification, explain whether linear transformations with the following properties can exist or not. In case such a linear transformation may exist, give an example for the same. [10]

   1. $T : \mathbb{R}^5 \to \mathbb{R}^4$ such that $ker(T) = R(T)$.
   2. $T : \mathbb{R}^4 \to \mathbb{R}^4$ such that $ker(T) = R(T)$.

9. Verify whether $\langle . , . \rangle : \mathbb{R}^2 \to \mathbb{R}$ defined by $\langle (x_1, y_1), (x_2, y_2) \rangle = x_1y_1 + x_2y_1 + x_1y_2 + x_2y_2$ is an inner product or not. Using this inner product, verify whether $\{(1, 0), (0, 1)\}$ is an orthonormal set or not. [10]

10. Obtain the Fourier series expansion of the periodic function $f(x) = e^x$, $-\pi < x < \pi$, $f(x + 2\pi) = f(x)$. Hence find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{1 + n^2}$. [10]

11. Find the Fourier cosine transform of $f(x) = e^{-x^2}$. [10]

12. Find the first, second and third level Haar transform for the signal (56, 40, 8, 24, 48, 48, 40, 16). [10]

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